Intermediate ALGEBRA

with POWER Learning

Sherri Messersmith
Lawrence Perez
Robert S. Feldman

What is POWER?

P.O.W.E.R. is an approach to systematically completing tasks based on five practical steps

P Prepare

The most critical facet of preparation is setting goals. Goal-setting improves student performance by increasing student focus, confidence, motivation, and persistence.

O Organize

Students have to identify and organize the intellectual tools necessary to accomplish their goals. Organizing refers not only to considering how they must apply the most appropriate academic strategies, but also to maintaining and applying good habits outside of class in order to manage their many responsibilities.

W Work

Doing the work–reading the materials, taking good notes in class, and doing in-class exercises–may seem like the most obvious step, but it is an area where students often falter. Using P.O.W.E.R. will improve your students' motivation and help them view success as a product of their hard work and effort.

E Evaluate

In math, concepts build on each other, so student success depends on reaching a level of mastery in each section before progressing. P.O.W.E.R. helps students understand that their work is not complete until they have assessed their progress and identified where they are struggling.

R Rethink

Too often in developmental math, students do not stop to assess their overall performance until after an exam, at which point it may be too late. P.O.W.E.R. prompts students after each section to honestly assess how they are doing and where they may need to change their strategy or ask for help.

Why Math Needs POWER

The P.O.W.E.R. series is a solution for students struggling to achieve their academic goals and instructors trying to balance teaching math with teaching effective student behavior.

Sherri Messersmith

Professor of Mathematics, College of DuPage

"Developmental students don't *want* to fail, they just don't know how to succeed. Teaching students at the developmental level is not just about teaching mathematics; it's about helping them learn all of the 'other' things it takes to be successful in college (and in life). If students don't know how to read a textbook, how to manage their time, how to take notes, how to set realistic goals, how to study, etc.—if they don't know how to be college students, then how can they be successful in our math courses? This challenge for students becomes our challenge as instructors. How do we teach successful student behavior while also getting through the objectives in our math courses?"

"**First and foremost, this is a math textbook.** Its purpose is to present all of the mathematics that is required for a course in the clearest, most mathematically precise way possible through the use of both traditional methods and some that you will find unique. But we think it is crucial to address study strategies as well, and, in fact, Larry and I wrote the manuscripts this way even before we met Bob! We teamed up to formalize the steps by integrating the P.O.W.E.R. framework into the textbooks, customizing it for teaching and learning mathematics. It turned out to be a very natural fit."

Lawrence Perez

Professor of Mathematics, Saddleback College

"It's not just about the content. Having an easy-to-read text coupled with thought-provoking pedagogy, in many cases, is not enough. As instructors, we strive to teach our students how to be successful learners. Integrating study strategies that are not directly connected to the text may not suffice. This is what makes the P.O.W.E.R. Math Series different. Dr. Feldman's research has provided the necessary framework that can assist developmental math students in navigating the difficult transition from arithmetic to abstract algebra and it is applicable to a variety of learning environments. It is also a framework that we as developmental math instructors can use to create our own supplemental resources and refine our personal teaching styles."

Bob Feldman

Dean and Professor of Psychology, University of Massachusetts Amherst,
Director of POWER Up for Student Success, The UMass first-year experience program

"Good students are made, not born. What does being a 'good' student at the college level mean? Among other things, it means coming to class prepared, taking good notes, developing good study habits, and managing time effectively. These skills can be taught, and taught in ways that lead to success. In fact, careful research shows courses that cover the skills related to college success produce a significant and demonstrable rise in student retention beyond the first semester in college. After years of teaching and doing research in student success, I developed the P.O.W.E.R. Learning framework to help students to achieve their goals in any class and even after they graduate."

"A developmental math course is often one of the first courses students take at the college level, and it is the gateway to continuing their college education. We have integrated a proven method of helping students develop basic skills into a text written by instructors with years of classroom experience."

What Students Are Saying about

POWER

Dozens of college students participated in focus groups to help develop this series. Here is what they told us. To see video clips of students talking about P.O.W.E.R. go to www.mhhe.com/sem/powermath

"In math class, I usually dive head first into the problems, hoping the material will come to me. I don't evaluate my work which leads to dishonesty about my strengths and weaknesses and whether I understand the concepts enough to move on. **The "Evaluate" step in P.O.W.E.R. will really help improve my grades."**

– Ashley Grayson, *Northeastern Illinois University*

"P.O.W.E.R. will help prepare me to do or learn something that is a little challenging. **It helps me identify my goals and what I need to do to get there."**

– Nina Turnage, *Wilbur Wright College*

"I always work in a trial and error kind of way. P.O.W.E.R. will help me study more efficiently because **it breaks down learning into steps."**

– Zainab Khomusi, *University of Illinois at Chicago*

"The P.O.W.E.R. framework is like the scientific method for learning. **Even outside of class, I think it will help me be successful in business and life goals."**

– Nathan Hurde, *University of Illinois at Chicago*

"You can use **P.O.W.E.R. to attack learning math,** or really any subject, successfully."

– Eire Aatnite, *Roosevelt University*

"P.O.W.E.R. would motivate and guide me to take more time with math and **recognize and fix my weaknesses."**

– Jaimie O'Leary, *Northeastern Illinois University*

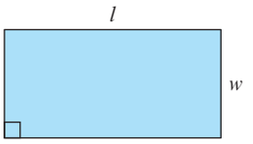
"I like that this framework already fits in with a lot of my study habits. **It just makes sense."**

– Lauren Mosley, *Western Illinois University*

Geometry Formulas

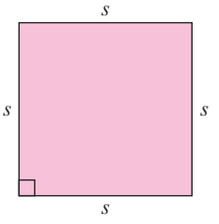
Rectangle

Perimeter: $P = 2 \cdot l + 2 \cdot w$
Area: $A = l \cdot w$



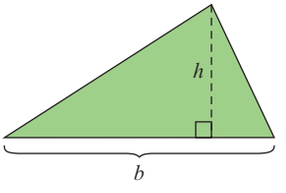
Square

Perimeter: $P = 4 \cdot s$
Area: $A = s^2$



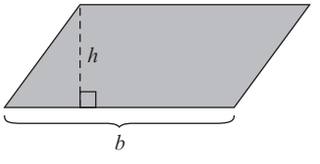
Triangle

Area: $A = \frac{1}{2} \cdot b \cdot h$



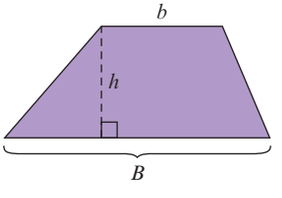
Parallelogram

Area: $A = b \cdot h$



Trapezoid

Area: $A = \frac{1}{2} \cdot h \cdot (b + B)$



Circle

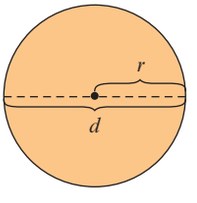
Diameter: $d = 2 \cdot r$

Radius: $r = \frac{d}{2}$

Circumference: $C = \pi \cdot d$
$C = 2 \cdot \pi \cdot r$

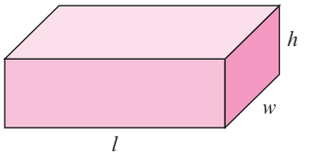
Area: $A = \pi \cdot r^2$

$\pi \approx 3.14$ or $\pi \approx \frac{22}{7}$



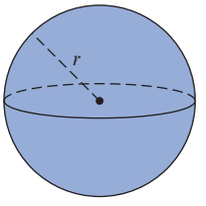
Rectangular Solid

Volume: $V = l \cdot w \cdot h$



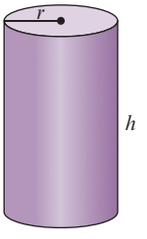
Sphere

Volume: $V = \frac{4}{3} \cdot \pi \cdot r^3$



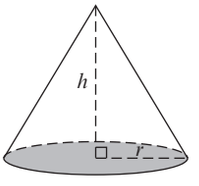
Cylinder

Volume: $V = \pi \cdot r^2 \cdot h$



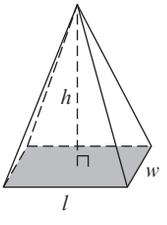
Cone

Volume: $V = \frac{1}{3} \cdot \pi \cdot r^2 \cdot h$

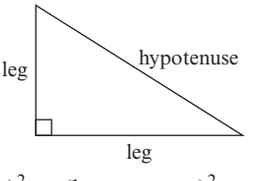


Pyramid

Volume: $V = \frac{1}{3} \cdot l \cdot w \cdot h$



Right Triangle



$(\text{leg})^2 + (\text{leg})^2 = (\text{hypotenuse})^2$

$\text{leg} = \sqrt{(\text{hypotenuse})^2 - (\text{known leg})^2}$

Intermediate

ALGEBRA

INTERMEDIATE ALGEBRA WITH P.O.W.E.R. LEARNING

This book is printed on acid-free paper.

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About the Authors

Sherri Messersmith

Professor of Mathematics, College of DuPage

Sherri Messersmith began teaching at the College of DuPage in Glen Ellyn, Illinois in 1994 and has over 25 years of experience teaching many different courses from developmental mathematics through calculus. She earned a Bachelor of Science degree in the Teaching of Mathematics at the University of Illinois at Urbana-Champaign and taught at the high school level for two years. Sherri returned to UIUC and earned a Master of Science in Applied Mathematics and stayed on at the university to teach and coordinate large sections of undergraduate math courses as well as teach in the Summer Bridge program for at-risk students. In addition to the P.O.W.E.R. Math Series, she is the author of a hardcover series of textbooks and has also appeared in videos accompanying several McGraw-Hill texts.

Sherri and her husband are recent empty-nesters and live in suburban Chicago. In her precious free time, she likes to read, cook, and travel; the manuscripts for her books have accompanied her from Spain to Greece and many points in between.

Lawrence Perez

Professor of Mathematics, Saddleback College

Larry Perez has fifteen years of classroom experience teaching math and was the recipient of the 2010 Community College Professor of the Year Award in Orange County, California. He realized early on that students bring to the classroom different levels of attitude, aptitude, and motivation sometimes accompanied by a tremendous fear of taking math. Confronted by this, he developed a passion for engaging students, demanding him to innovate traditional and online pedagogical techniques using architecture created with student feedback as the mechanism of design. He is the creator of the award-winning online learning environment Algebra2go® and has presented his work and methodology at conferences around the country.

Larry is a Veteran of the United States Navy Submarine Force and is a graduate of California State University Fullerton earning degrees in Electrical Engineering and Applied Mathematics. In his spare time he enjoys mountain biking and the great outdoors.

Robert S. Feldman

Dean and Professor of Psychology, University of Massachusetts Amherst

Bob Feldman still remembers those moments of being overwhelmed when he started college at Wesleyan University. "I wondered whether I was up to the challenges that faced me," he recalls, "and although I never would have admitted it then, I really had no idea what it took to be successful at college."

That experience, along with his encounters with many students during his own teaching career, led to a life-long interest in helping students navigate the critical transition that they face at the start of their own college careers. Bob, who went on to receive a doctorate in psychology from the University of Wisconsin-Madison, teaches at the University of Massachusetts Amherst, where he is the Dean of the College of Social and Behavioral Sciences and Professor of Psychology. He also directs a first-year experience course for incoming students.

Bob is a Fellow of both the American Psychological Association and the Association for Psychological Science. He has written more than 200 scientific articles, book chapters, and books, including P.O.W.E.R. Learning: *Strategies for Success in College and Life,* 6e and *Understanding Psychology,* 11e. He is president-elect of the FABBS Foundation, an umbrella group of societies promoting the behavioral and brain sciences.

Bob loves travel, music, and cooking. He and his wife live near the Holyoke mountain range in western Massachusetts.

Table of Contents

Consistent Integration of Study Skills

In *Intermediate Algebra,* strategies for learning are presented alongside the math content, making it easy for students to learn math *and* study skills at the same time. The P.O.W.E.R. framework aligns with the math learning objectives, providing instructors with a resource that has been consistently integrated throughout the text.

A **STUDY STRATEGIES** feature begins each chapter. Utilizing the P.O.W.E.R. framework, these boxes present steps for mastering the different skills students will use to succeed in their developmental math course. For example, these boxes will contain strategies on time management, taking good notes and, as seen in the sample below, taking a math test.

POWER

Study Strategies Taking Math Tests

You've studied your class notes and your textbook. You've reviewed your homework and problem sets. You've even worked with your fellow students in a study group. Now it's time to sit down and actually take the test you've prepared for. When this moment co don't you owe it to yourself to perform your best? Use the following strategies to e that you get the grade you deserve.

P Prepare

- Study smart. Focus your preparation on the material that will be covered on t test. The emPOWERme on page 281 will help you.
- Practice, practice, practice! Repetition will help build your skills. Do problems using a timer to simulate the amount of time you will have to take the test.
- Get a good night's sleep before the test, as it is during sleep that short-term memory turns into long-term memory.
- Warm up for the test just like athletes do before they play a game. No matter much you studied the night before, do several "warm-up" problems the same as the test. This way, you will be in the groove of doing math and won't go int the test cold.

O Organize

- Arrive at the test location early.
- Have multiple pens, pencils, scratch paper, and your textbook with you when y sit down to take the test.
- Warm-up your brain by reviewing your notes and working through problems those that will appear on the test.

W Work

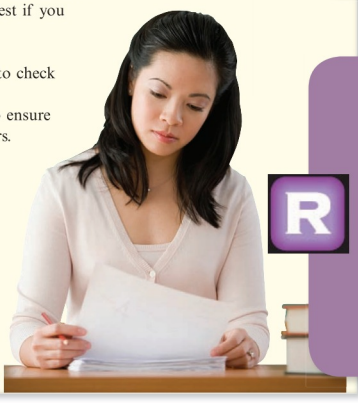
- Before you start the test, read all the instructions.
- Answer the easiest questions first, leaving time at the end to work on h problems.
- Show all your work in a neat and organized way because your instructor may you credit for this even if you don't finish the problem.
- Remain calm throughout the test. Take a deep breath and focus on doing your best if you start to feel anxious.

E Evaluate

- Leave time at the end of the test to check over your work.
- Double-check your calculations to ensure you didn't make any careless errors.

R Rethink

- Review your test carefully when you get it back. You want to understand the source of any errors you made.
- Consider the process you used to study for the test, and think about ways it could be improved.



...all the work you do throughout the term—the note-taking, the homework, all of it—represents test preparation. The strategies below will help you perform your best on math tests.

W

...answer the easiest questions first. This will build your confidence and leave you more time to work on the harder problems.

R

...when you get your test back... look it over and see where you made errors. Remember, math concepts build on each other—you can't master the new concepts until you are comfortable with the previous ones.

CHAPTER AND SECTION POWER PLANS Before getting started on reading the chapter, a student will focus on Preparation and Organization skills in the POWER **Plans.** These tools give practical suggestions for setting and achieving goals. The steps revolve around best practices for student success and then apply P.O.W.E.R. toward learning specific concepts in math.

Chapter 5 POWER Plan

P Prepare	O Organize
What are your goals for Chapter 5?	**How can you accomplish each goal?**
1 Be prepared before and during class.	• Don't stay out late the night before, and be sure to set your alarm clock! • Bring a pencil, notebook paper, and textbook to class. • Avoid distractions by turning off your cell phone during class. • Pay attention, take good notes, and ask questions. • Complete your homework on time, and ask questions on problems you do not understand. • Plan ahead for tests by preparing many days in advance.
2 Understand the homework to the point where you could do it without needing any help or hints.	• Read the directions, and show all of your steps. • Go to the professor's office for help. • Rework homework and quiz problems, and find similar problems for practice. • Review old material that you have not mastered yet.
3 Use the P.O.W.E.R. framework to help you take tests: *Studying Smart.*	• Read the Study Strategy that explains how to study effectively for tests. • Do a "practice run" the night before the test by doing a practice test without notes. • Complete the emPOWERme that appears before the Chapter Summary.
4 Write your own goal. _____ _____	• _____ _____
What are your objectives for Chapter 5?	**How can you accomplish each objective?**
1 Be able to solve a system of linear equations in two variables by using the graphing, substitution, or elimination methods. Know when to use each method.	• Learn the procedures for each of these methods. • Know the terminology associated with the solutions such as independent and consistent. • Know how to check each answer.
2 Be able to determine when the solution to a system of equations is *no solution* or *infinite solutions*. Know what these solutions look like on a graph and how to write the answer.	• Learn the procedures for solving a system of equations and the possible answers when variables "drop out." • Learn the terminology associated with the solutions such as *inconsistent* and *dependent*. • Know what these results look like on a graph. • Know how to check your solutions.
3 Be able to solve a system of linear equations in three variables, including systems where there are missing terms.	• Learn the procedure for **Solving a System of Linear Equations in Three Variables.** • Know how to check your solutions.

Hint
Be sure you are writing out each step as you are reading the example.

b) To solve $t^2 - 20 = 0$, begin by getting t^2 on a side by itself.

$$\begin{aligned} t^2 - 20 &= 0 \\ t^2 &= 20 && \text{Add 20 to each side.} \\ t &= \pm\sqrt{20} && \text{Square root property} \\ t &= \pm\sqrt{4}\cdot\sqrt{5} && \text{Product rule for radicals} \\ t &= \pm 2\sqrt{5} && \sqrt{4} = 2 \end{aligned}$$

Check:

$t = 2\sqrt{5}$:
$t^2 - 20 = 0$
$(2\sqrt{5})^2 - 20 \stackrel{?}{=} 0$
$(4 \cdot 5) - 20 \stackrel{?}{=} 0$
$20 - 20 = 0$ ✓

$t = -2\sqrt{5}$:
$t^2 - 20 = 0$
$(-2\sqrt{5})^2 - 20 \stackrel{?}{=} 0$
$(4 \cdot 5) - 20 \stackrel{?}{=} 0$
$20 - 20 = 0$ ✓

The solution set is $\{-2\sqrt{5}, 2\sqrt{5}\}$.

WORK HINTS provide additional explanation and point out common places where students might go wrong when solving a problem. Along with the ***Be Careful*** boxes, these tools act as a built-in tutor, helping students navigate the material and learn concepts even outside of class.

IN-CLASS EXAMPLES are available only in the Annotated Instructor Edition. These examples offer instructors additional problems to work through in class. In-class example problems align with the Guided Student Notes resource available with this package.

EXAMPLE 1 Solve using the square root property.

a) $x^2 = 9$ b) $t^2 - 20 = 0$ c) $2a^2 + 21 = 3$

Solution

a)
$$\begin{aligned} & x^2 = 9 \\ & \swarrow \quad \searrow \\ & x = \sqrt{9} \quad \text{or} \quad x = -\sqrt{9} \qquad \text{Square root property} \\ & x = 3 \quad \text{or} \quad x = -3 \end{aligned}$$

The solution set is $\{-3, 3\}$. The check is left to the student.

An equivalent way to solve $x^2 = 9$ is to write it as

$$\begin{aligned} x^2 &= 9 \\ x &= \pm\sqrt{9} && \text{Square root property} \\ x &= \pm 3 \end{aligned}$$

The solution set is $\{-3, 3\}$. We will use this approach when solving equations using the square root property.

PUTTING IT ALL TOGETHER One of the challenges students struggle with is putting all of the steps they've learned together and *applying* that knowledge to a problem. ***Putting It All Together*** sections will help students understand the big picture and work through the toughest challenge when solving applications—*problem recognition,* or knowing *when* to use *what* method or thought process. These sections include a summary and several problems that help students reason through a problem using conversational, yet mathematically correct, language.

Putting It All Together

Prepare | **Organize**

What are your objectives for Putting It All Together?	How can you accomplish each objective?
1 Decide Which Method to Use to Solve a Quadratic Equation	• Be able to write out all the different methods in your notes. • Review characteristics of each method, and be able to identify the most efficient method for each problem. • Try solving some problems using more than one method, if time permits, and check your answers. • Complete the given example on your own. • Complete You Try 1.

Work Read the explanations, follow the examples, take notes, and complete the You Try.

emPOWERme Studying Smart

Imagine packing your bag for a trip without knowing where you were going. Would you take sweaters or swimsuits? A raincoat or sunblock? This dilemma is parallel to the one you face if you study for a test without knowing what it will cover. The key to effective studying—and to successful test-taking—is to tailor your efforts to the test you will have to take. Before you start studying, answer these questions, and use your answers to help you prepare.

- Is the test called a "test," "exam," "quiz," or something else? There is a difference! Exams tend to be longer, while quizzes are often shorter and narrower in their focus. If you are aren't sure, ask your instructor.
- What material will the test cover? Will it cover only the most recent subjects or everything you've learned in the term so far?
- How many questions will be on the test? How much time is it expected to take? A full class period? Only part of a period?
- What kinds of questions will be on the test?
- Will you be allowed to use a calculator? Consult your textbook?
- Will the test be graded on a curve?
- Will sample questions be provided?
- Are tests from previous terms available for you to study?
- How much does the test contribute to your final course grade?

emPOWERme boxes circle back to the opening **Study Strategies** and give students a checklist to evaluate how well they followed through on all of the positive habits recommended to successfully master a skill.

Instructor POWER Tool Kit

The Messersmith/Perez/Feldman Series offers instructors a robust digital resources package to help you with all of your teaching needs.

Resources in your P.O.W.E.R. tool kit include:

- Connect Hosted by Aleks*
- ALEKS 360*
- Instructor Solutions Manual
- Student Solutions Manual
- Guided Student Notes*
- Classroom Worksheets*
- Instructor Resource Manual
- Test Bank Files
- Computerized Test Bank
- Faculty Development and Digital Training*
- PowerPoint Presentations
- Extensive Video Package*

***Details of these resources are included in the following pages!**

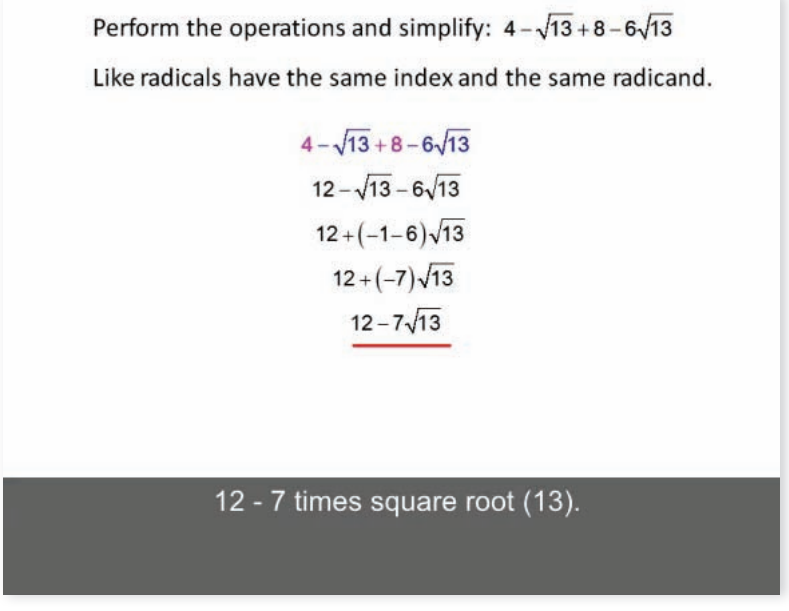
Videos

Hundreds of videos are available to guide students through the content, offering support and instruction even outside your classroom.

Exercise Videos – These 3–5-minute clips show students how to solve various exercises from the textbook. With around thirty videos for every chapter, your students are supported even outside the classroom.

Lecture Videos – These 5–10-minute videos walk students through key learning objectives and problems from the textbook.

P.O.W.E.R. Videos – These engaging segments guide students through the P.O.W.E.R. framework and the study skills for each chapter.



Faculty Development and Digital Training

McGraw-Hill is excited to partner with our customers to ensure success in the classroom with our course solutions.

Looking for ways to be more effective in the classroom? Interested in learning how to integrate student success skills in your developmental math courses?

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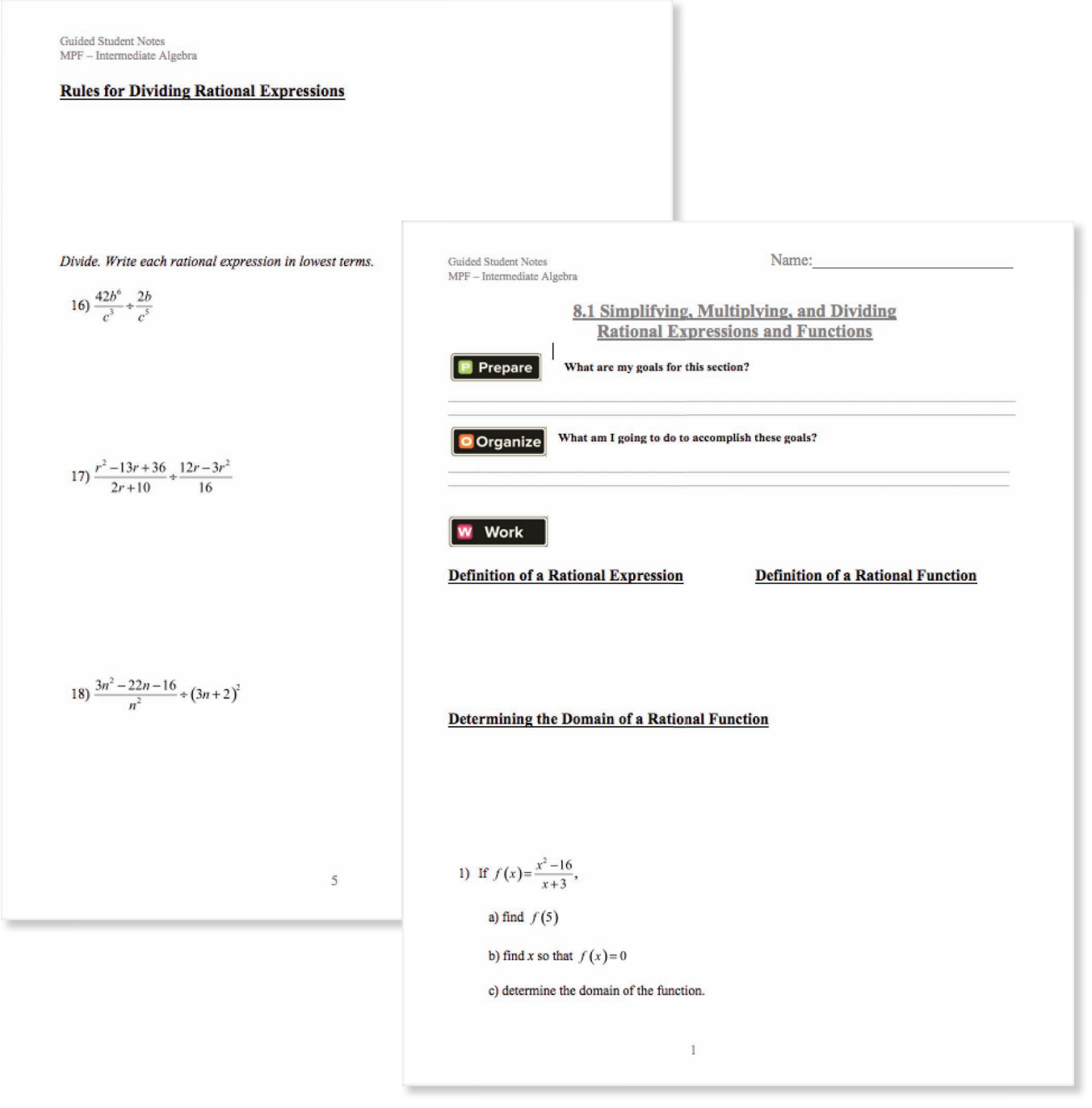
Need a tool to help your students take better notes?

GUIDED STUDENT NOTES

By taking advantage of Guided Student Notes, your students will have more time to learn the material and participate in solving in-class problems while, at the same time, becoming better note takers. Ample examples are included for appropriate coverage of a topic that will not overwhelm students. Use them as they are or download and edit the Guided Student Notes according to your teaching style.



Guided Student Notes
MPF – Intermediate Algebra

Rules for Dividing Rational Expressions

Divide. Write each rational expression in lowest terms.

16) $\frac{42b^6}{c^3} \div \frac{2b}{c^5}$

17) $\frac{r^2-13r+36}{2r+10} \div \frac{12r-3r^2}{16}$

18) $\frac{3n^2-22n-16}{n^2} \div (3n+2)^2$

5

Guided Student Notes
MPF – Intermediate Algebra

Name:_____

8.1 Simplifying, Multiplying, and Dividing Rational Expressions and Functions

Prepare — What are my goals for this section?

Organize — What am I going to do to accomplish these goals?

Work

Definition of a Rational Expression

Definition of a Rational Function

Determining the Domain of a Rational Function

1) If $f(x)=\frac{x^2-16}{x+3}$,

a) find $f(5)$

b) find x so that $f(x)=0$

c) determine the domain of the function.

1

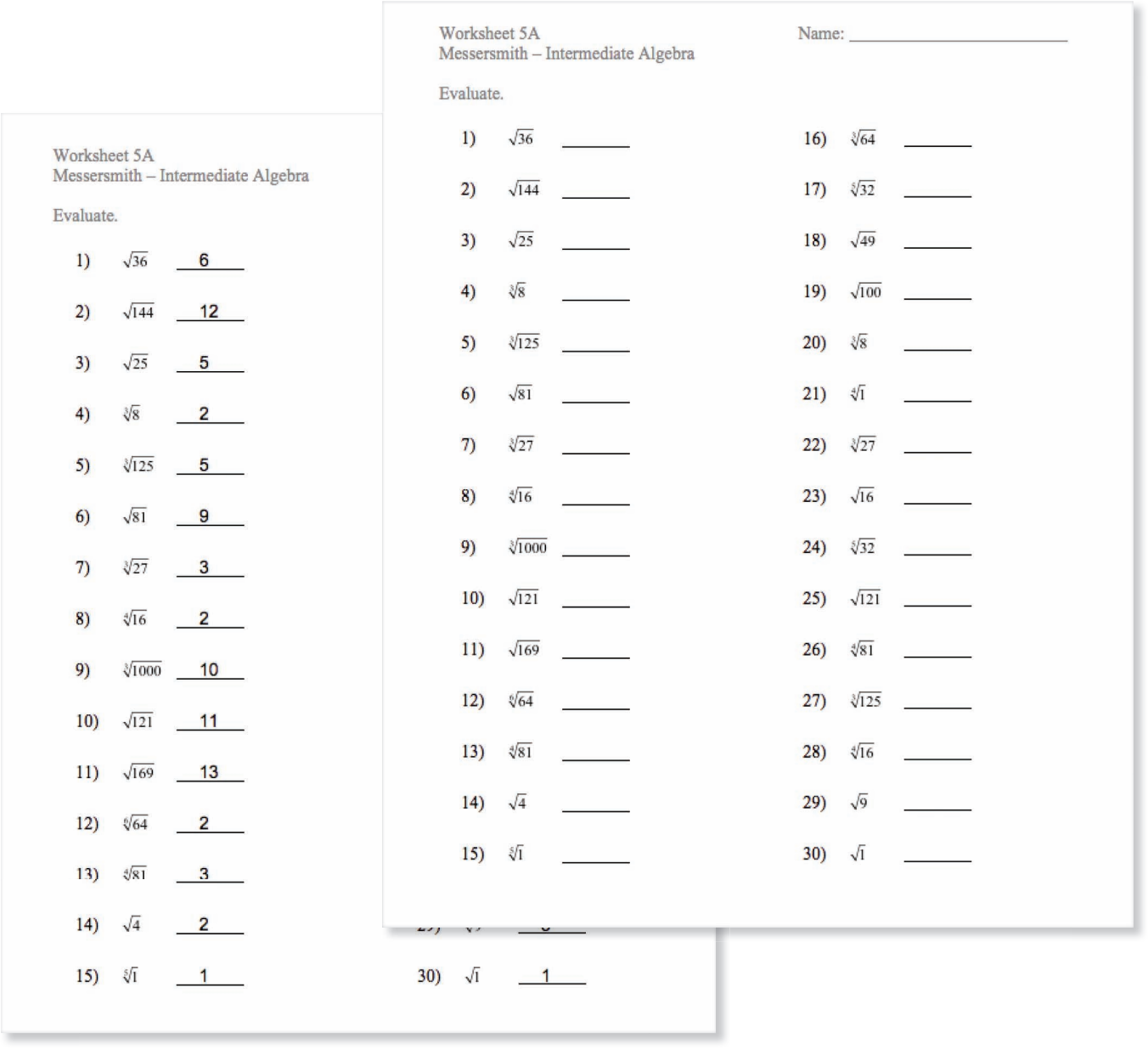
Develop your students' basic skills with a ready-made resource.

WORKSHEETS FOR STUDENT AND INSTRUCTOR USE

Worksheets for every section are available as an instructor supplement. These author-created worksheets provide a quick, engaging way for students to work on key concepts. They save instructors from having to create their own supplemental material and address potential stumbling blocks in student understanding. Classroom tested and easy to implement, they are also a great resource for standardizing instruction across a mathematics department.

The worksheets fall into three categories: Worksheets to Improve Basic Skills; Worksheets to Help Teach New Concepts; and Worksheets to Tie Concepts Together.

The worksheets are available in an instructor edition, with answers, and in a student edition, without answers.



Worksheet 5A
Messersmith – Intermediate Algebra

Evaluate.

1) $\sqrt{36}$ 6
2) $\sqrt{144}$ 12
3) $\sqrt{25}$ 5
4) $\sqrt[3]{8}$ 2
5) $\sqrt[3]{125}$ 5
6) $\sqrt{81}$ 9
7) $\sqrt[3]{27}$ 3
8) $\sqrt[4]{16}$ 2
9) $\sqrt[3]{1000}$ 10
10) $\sqrt{121}$ 11
11) $\sqrt{169}$ 13
12) $\sqrt[6]{64}$ 2
13) $\sqrt[4]{81}$ 3
14) $\sqrt{4}$ 2
15) $\sqrt[5]{1}$ 1
30) $\sqrt{1}$ 1

Worksheet 5A
Messersmith – Intermediate Algebra

Name: _____

Evaluate.

1) $\sqrt{36}$ _____
2) $\sqrt{144}$ _____
3) $\sqrt{25}$ _____
4) $\sqrt[3]{8}$ _____
5) $\sqrt[3]{125}$ _____
6) $\sqrt{81}$ _____
7) $\sqrt[3]{27}$ _____
8) $\sqrt[4]{16}$ _____
9) $\sqrt[3]{1000}$ _____
10) $\sqrt{121}$ _____
11) $\sqrt{169}$ _____
12) $\sqrt[6]{64}$ _____
13) $\sqrt[4]{81}$ _____
14) $\sqrt{4}$ _____
15) $\sqrt[5]{1}$ _____
16) $\sqrt[6]{64}$ _____
17) $\sqrt[5]{32}$ _____
18) $\sqrt{49}$ _____
19) $\sqrt{100}$ _____
20) $\sqrt[3]{8}$ _____
21) $\sqrt[5]{1}$ _____
22) $\sqrt[3]{27}$ _____
23) $\sqrt{16}$ _____
24) $\sqrt[5]{32}$ _____
25) $\sqrt{121}$ _____
26) $\sqrt[4]{81}$ _____
27) $\sqrt[3]{125}$ _____
28) $\sqrt[4]{16}$ _____
29) $\sqrt{9}$ _____
30) $\sqrt{1}$ _____

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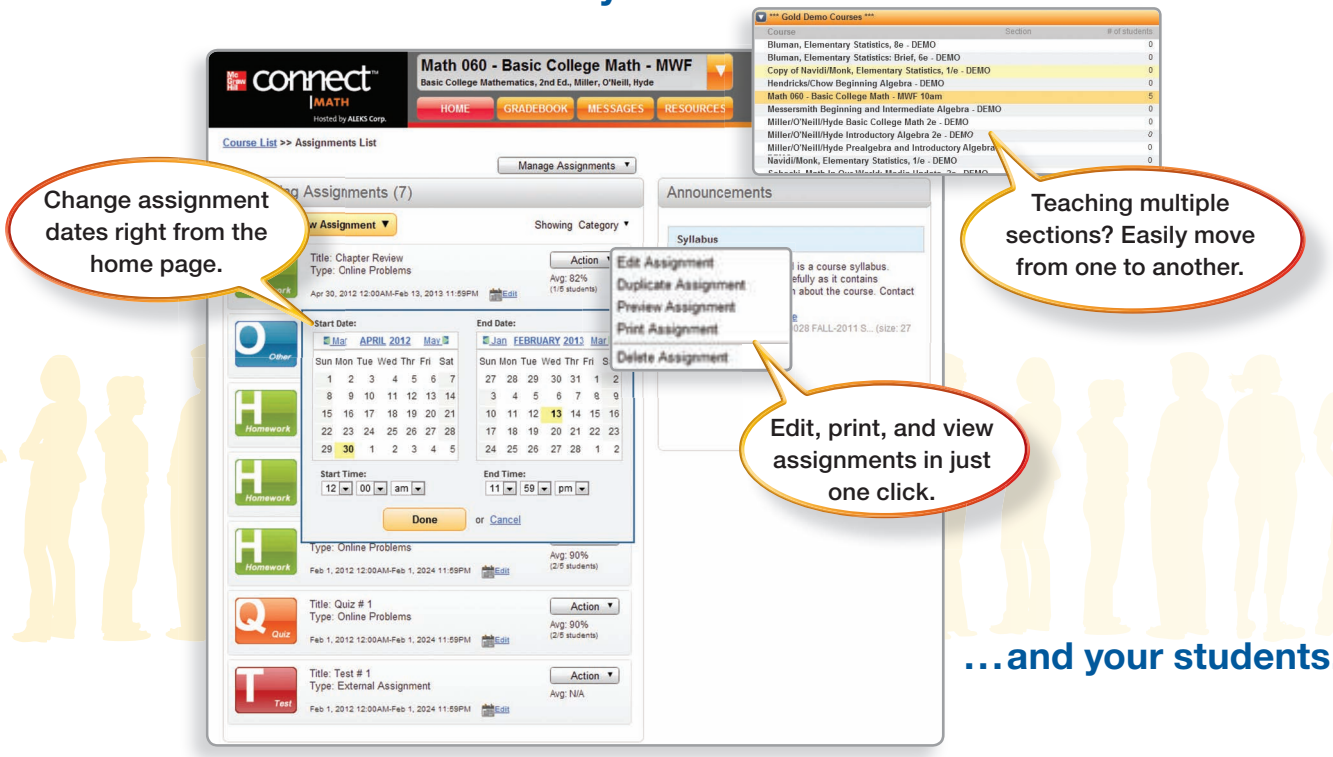
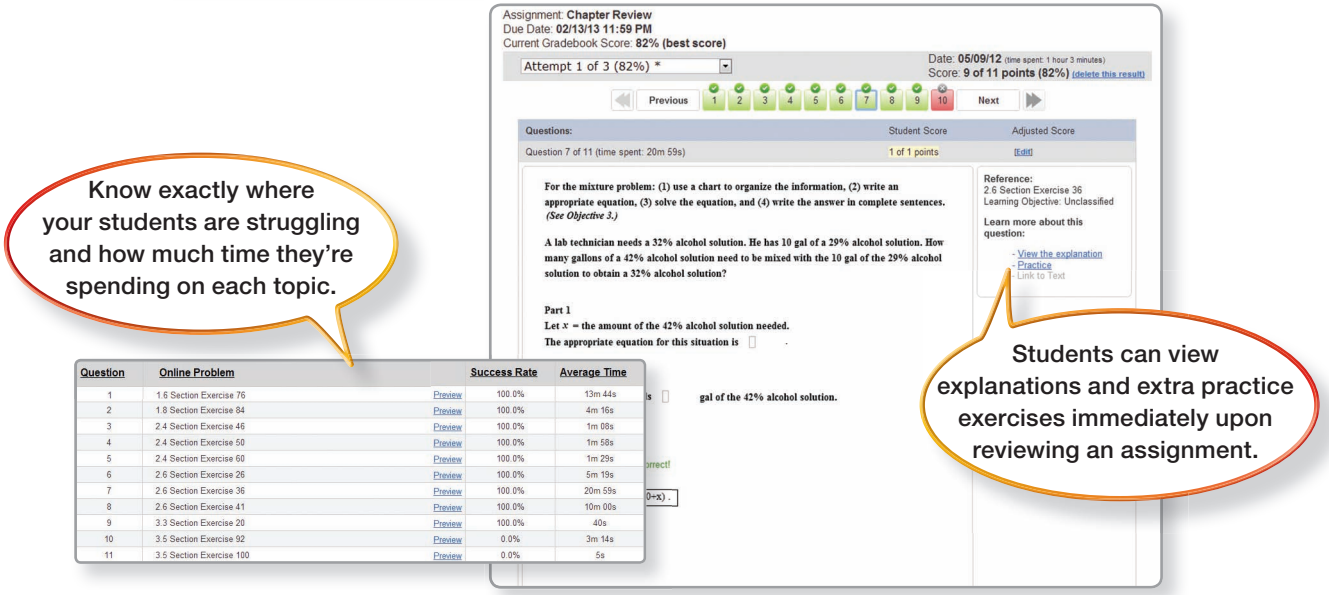


…and your students.



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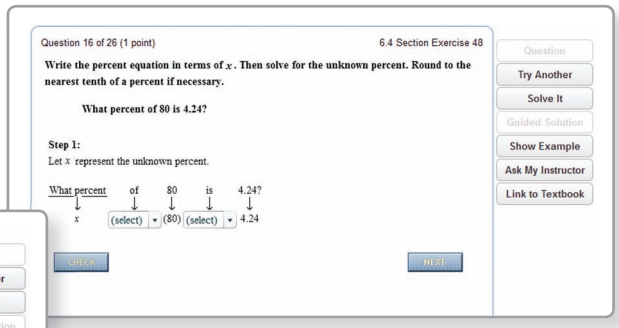
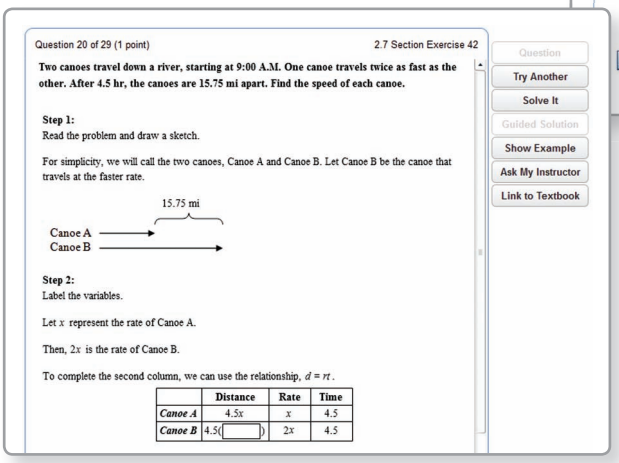
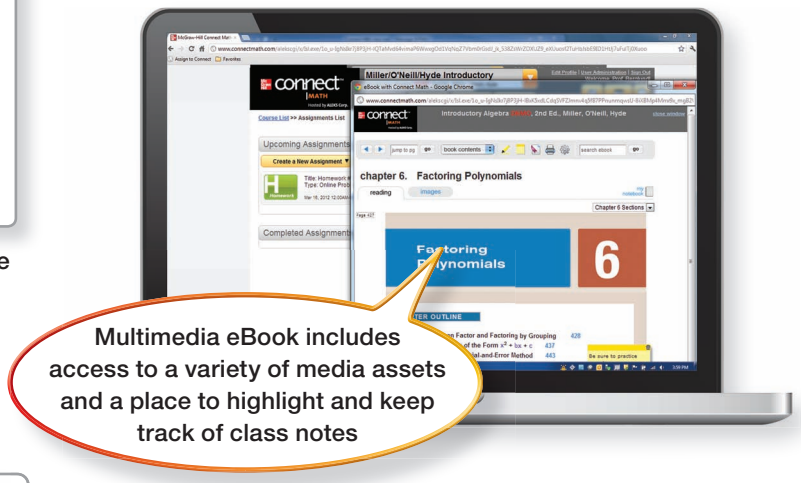
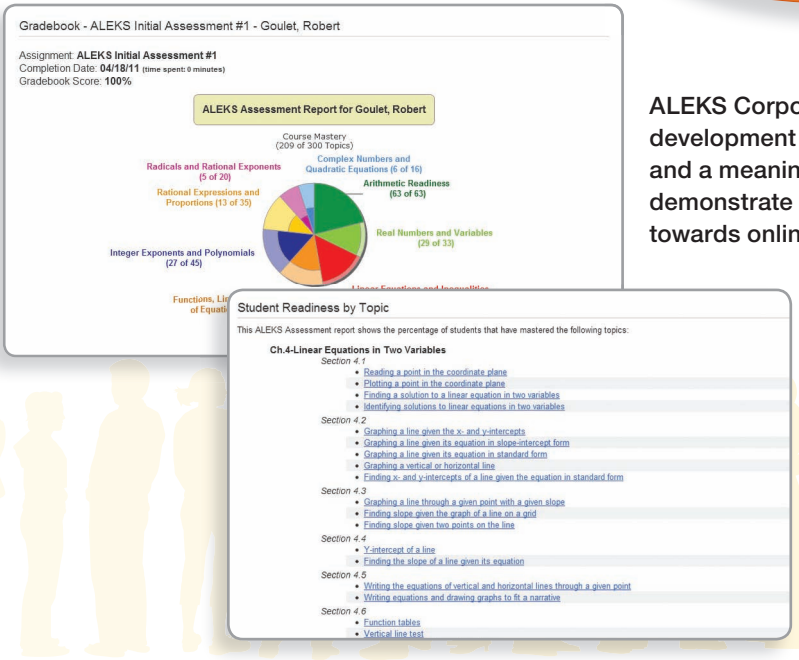




For consistency, the guided solutions match the style and voice of the original text as though the author is guiding the students through the problems.



Multimedia eBook includes access to a variety of media assets and a place to highlight and keep track of class notes



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The ALEKS® Initial Assessment is an artificially intelligent (AI), diagnostic assessment that identifies precisely what a student knows. Instructors can then use this information to make more informed decisions on what topics to cover in more detail with the class.

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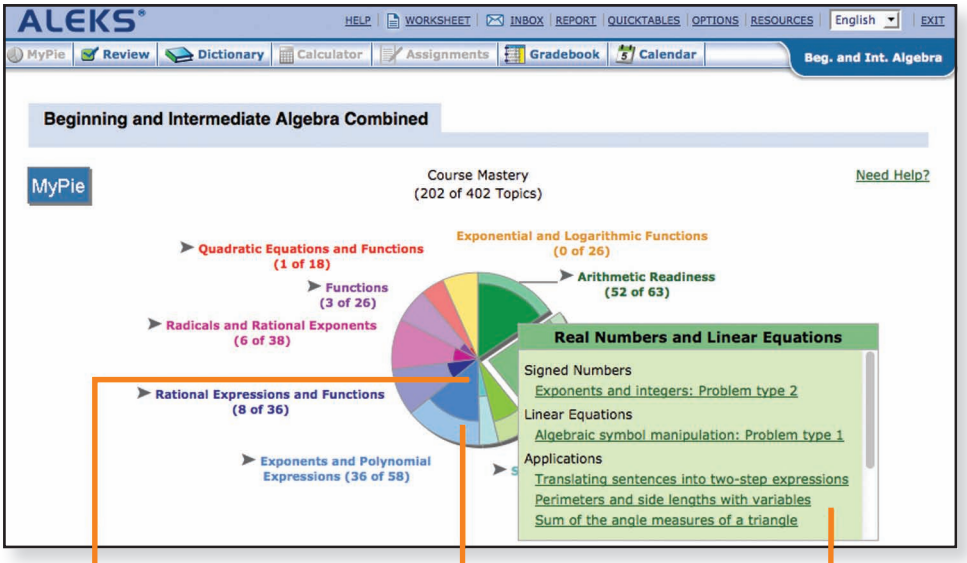

The ALEKS Pie summarizes a student's current knowledge, then delivers an individualized learning path with the exact topics the student is most ready to learn.

Dark portion represents what the student knows.

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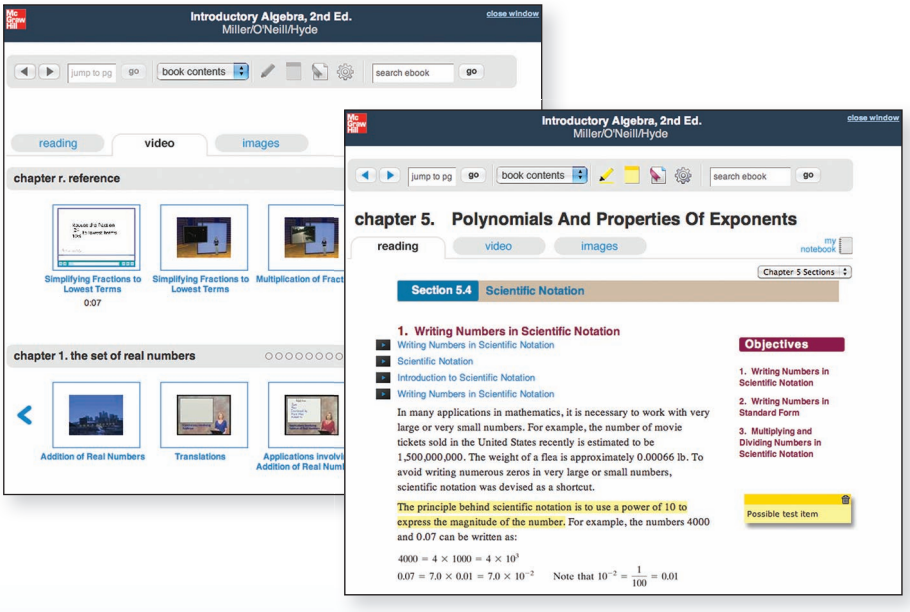

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SOCIOLOGY AND DEMOGRAPHICS

SPORTS AND HOBBIES

CHAPTER 1

Real Numbers and Algebraic Expressions

OUTLINE

Math at Work:

Computer Game Designer

Ever since he was a child, Dave Cantelmo has known what he wanted to do: create video games. "I still remember playing games on the classic systems I had growing up," Dave says. "I would spend hours and hours playing those games and always imagined the games I wanted to create myself one day."

In order to realize his ambitions, Dave put in the time and effort necessary to learn the design and computer programming skills involved in video game development. Early on in college, he was particularly focused on building up his math abilities, as math is critical to the technical side of video game creation. "I was always a little intimidated by math," Dave describes. "But with hard work and the help of my instructors, I was able to turn math into one of my strengths. Now, it's something I use in my job every day."

Creating a successful video game is a complex challenge. "Completing a video game takes the coordinated efforts of dozens of designers, writers, programmers, animators, and many other people, all working for years to turn an idea into a game people all over the world can play," Dave explains. He says the key to completing such a difficult task is taking a smart, organized approach, dividing the work into steps that will culminate in the finished product.

In this chapter, we will discuss the topic of real numbers. We will also introduce P.O.W.E.R., a framework that can help you succeed in the complex challenges you face, either in the classroom, on the job, or in your daily life.

Study Strategies The P.O.W.E.R. Framework

The **P.O.W.E.R. Framework** is based on an acronym—a word formed from the first letters of a series of steps. P.O.W.E.R. stands for **P**repare, **O**rganize, **W**ork, **E**valuate, and **R**ethink. That's it. It's a simple framework, but an effective one. P.O.W.E.R. gives you a proven, ready-to-use approach to virtually any challenge you face, from studying for a math test to developing a presentation for your coworkers to writing the family grocery list. Think of its steps as a roadmap to success no matter what your task is.

Whether you are familiar with the P.O.W.E.R. framework already or are encountering it for the first time, take a moment now to review each of its steps in depth:

P Prepare

- Think about what you are trying to accomplish: Define both your short-term and long-term goals.
- **Long-term goals** are major accomplishments that take a significant amount of time and effort to achieve, such as graduating from college. **Short-term goals** are steps that are easier to accomplish and bring you closer to your long-term goals—for example, doing well on a math exam.

O Organize

- Identify the tools you will need to complete your task.
- Effective organization involves gathering both the *physical* tools you will need to complete your task (for example, a textbook, pen, paper, and so forth) and doing the *mental* work (reviewing lecture notes or major concepts in your textbook, say) to ensure you are ready to succeed.

W Work

- With the previous steps as your foundation, do the work of completing your task.
- When doing math tasks in particular, it is important to work efficiently but patiently, neither trying to rush through the work nor becoming frustrated and giving up after the first difficulty you encounter.
- Stay motivated by keeping your goals in mind.

E Evaluate

- Think back to your goal for the task. Did you meet your own expectations?
- Revise your work based on your assessment of it.
- In math courses, it's important to identify the specific obstacles that may be causing you to perform below your capabilities. Are there particular concepts you are struggling with? Would you benefit from working with a math tutor or your fellow students?

R Rethink

- Think critically about both the work you have done and the process you used to complete it. What did you do that worked well? Where do you see room for improvement?
- Take a step back and consider how the task you completed brought you closer to your long-term goals.

Chapter 1 POWER Plan

P Prepare

O Organize

What are your goals for Chapter 1?	How can you accomplish each goal?
1 Be prepared before and during class.	• Don't stay out late the night before and be sure to set your alarm clock! • Bring a pencil, notebook paper, and textbook to class. • Avoid distractions by turning off your cell phone during class. • Pay attention, take good notes, and ask questions. • Complete your homework on time and ask questions on problems you do not understand.
2 Understand the homework to the point where you could do it without needing any help or hints.	• Read the directions and show all of your steps. • Go to the professor's office for help. • Rework homework and quiz problems and find similar problems for practice.
3 Use the P.O.W.E.R. framework to help you organize your study: *Why Am I Going to College?*	• Read the Study Strategy that explains how to use P.O.W.E.R. • What does P.O.W.E.R. stand for? • Complete the emPOWERme that appears before the Chapter Summary.
4 Write your own goal. _____ _____	• _____ _____
What are your objectives for Chapter 1?	**How can you accomplish each objective?**
1 Learn the different sets of numbers.	• Learn the definitions in Section 1.1. • Take good notes in class.
2 Be able to add, subtract, multiply, and divide real numbers.	• Master the steps for these operations! Future sections build on this knowledge, so continually review these operations.
3 Understand how to evaluate expressions, roots, and exponents. Be able to simplify an expression using the order of operations.	• Understand the meaning of exponents and how to identify the base and exponent. • Memorize the powers of integers in Section 1.3. • Learn the **Order of Operations:** Use **P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally to help you remember the order of operations.
4 Understand the properties of real numbers and how they can be used to simplify problems.	• Understand the meaning of each property based on the meaning of the word. (Commutative property = "commute.") • Be aware of when the properties can be used to help simplify a problem.
5 Write your own goal. _____ _____	• _____ _____

W Work Read Sections 1.1 to 1.4, and complete the exercises.	
E Evaluate Complete the Chapter Review and Chapter Test. How did you do?	**R Rethink** • How did you perform on the goals for the chapter? Which steps could be improved for next time? If you had the chance to do this chapter over what would you do differently? • Think of a job you might like to have and describe how you would need to use what you have just learned to effectively do that job. • How has the P.O.W.E.R. framework helped you master the objectives of this chapter? Where else could you use this framework? Make it a point to use P.O.W.E.R. to complete another task this week.

1.1 Sets of Numbers

P Prepare **O Organize**

What are your objectives for Section 1.1?	How can you accomplish each objective?
1 Identify Numbers and Graph Them on a Number Line	• Know the definition of *natural numbers, whole numbers, integers, rational numbers,* and *irrational numbers.* • Determine when a number may belong to more than one number set. • Know how to draw/label a number line and determine where that number should be graphed. • Complete the given examples on your own. • Complete You Trys 1, 2, and 3.
2 Compare Numbers Using Inequality Symbols	• Write the inequality symbols in math notation and in words. • Know how to determine the larger of two given numbers. • Complete the given examples on your own. • Complete You Trys 4 and 5.
3 Find the Absolute Value of a Number	• Know the definitions of *additive inverse* and *absolute value.* • Be able to show, on a number line, the *additive inverse* and the *absolute value* of a given number. • Complete the given examples on your own. • Complete You Trys 6 and 7.

W Work **Read the explanations, follow the examples, take notes, and complete the You Trys.**

1 Identify Numbers and Graph Them on a Number Line

Why should we review sets of numbers and arithmetic skills? Because the manipulations done in arithmetic are precisely the same set of skills needed to learn algebra. Let's begin by defining some numbers used in arithmetic:

The set of **natural numbers** is {1, 2, 3, 4, 5, …}.

The set of **whole numbers** is {0, 1, 2, 3, 4, 5, …}.

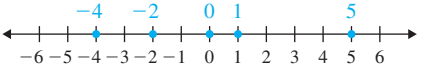
Natural numbers are often thought of as the counting numbers. Whole numbers consist of the natural numbers and zero. Let's look at other sets of numbers. We begin with integers. Remember that, on a number line, positive numbers are to the right of zero, and negative numbers are to the left of zero.

Definition

The set of **integers** includes the set of natural numbers, their negatives, and zero. The set of *integers* is {…, −3, −2, −1, 0, 1, 2, 3, …}.

EXAMPLE 1 Graph each number on a number line: 5, 1, −2, 0, −4.

Solution



5 and 1 are to the right of zero since they are positive. −2 is two units to the left of zero, and −4 is four units to the left of zero.

YOU TRY 1 Graph each number on a number line: 3, −1, 6, −5, −3.

Hint
Notice that we use the set of *integers* to label the number line in Example 1.

Positive and negative numbers are also called *signed numbers*.

EXAMPLE 2 Given the set of numbers $\left\{-11, 0, 9, -5, -1, \frac{2}{3}, 6\right\}$, list the

a) whole numbers b) natural numbers c) integers

Solution

a) whole numbers: 0, 6, 9 b) natural numbers: 6, 9

c) integers: −11, −5, −1, 0, 6, 9

YOU TRY 2 Given the set of numbers $\left\{3, -2, -9, 4, 0, \frac{5}{8}, -\frac{1}{3}\right\}$, list the

a) whole numbers b) natural numbers c) integers

Notice in Example 2 that $\frac{2}{3}$ did not belong to any of these sets. That is because the whole numbers, natural numbers, and integers do not contain any fractional parts. $\frac{2}{3}$ is a *rational number*.

Hint
Remember that whole numbers, natural numbers, and integers can all be written in the form $\frac{p}{q}$ where $q = 1$.

Definition

A **rational number** is any number of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

In other words, a rational number is any number that can be written as a fraction where the numerator and denominator are integers and the denominator does not equal zero.

Rational numbers include much more than numbers like $\frac{2}{3}$, which are already in fractional form. They also include numbers such as 3, 0.5, -7, $0.\overline{4}$, and the square root of 9 because each of these numbers can be written as a fraction.

To summarize, the set of rational numbers includes

1) integers, whole numbers, and natural numbers.
2) repeating decimals.
3) terminating decimals.
4) fractions and mixed numbers.

The set of rational numbers does *not* include nonrepeating, nonterminating decimals or radicals like $\sqrt{5}$ where the radicand is *not* a perfect square. These numbers cannot be written as the quotient of two integers. Numbers such as these are called *irrational numbers.*

Hint
Note that it is not possible to write an irrational number as a fraction. Some irrational numbers are π, $\sqrt{5}$, 0.412738. . .

Definition

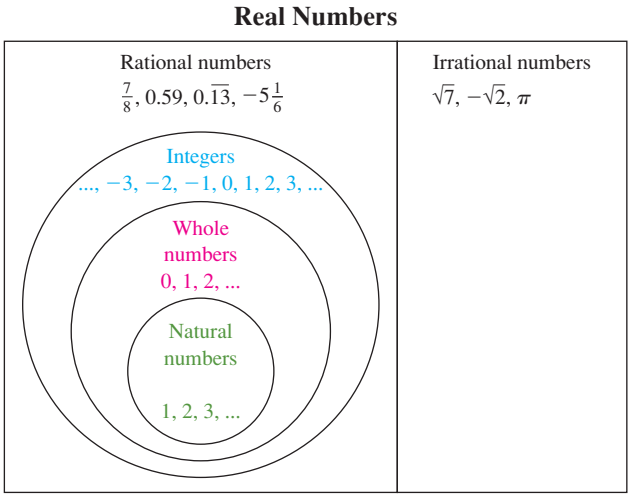
The set of numbers that cannot be written as the quotient of two integers is called the set of **irrational numbers.** Written in decimal form, an *irrational number* is a nonrepeating, nonterminating decimal.

If we put together the sets of numbers we have discussed up to this point, we get the *real numbers.*

Definition

The set of **real numbers** consists of the rational and irrational numbers.

We summarize the information next with examples of the different sets of numbers:



From the figure we can see, for example, that all whole numbers {0, 1, 2, 3, …} are integers, but not all integers are whole numbers (−3, for example).

EXAMPLE 3

Given the set of numbers $\left\{0.\overline{2}, 37, -\frac{4}{15}, \sqrt{11}, -19, 8.51, 0, 6.149235...\right\}$, list the

a) integers b) natural numbers c) whole numbers

d) rational numbers e) irrational numbers f) real numbers

Solution

a) integers: −19, 0, 37

b) natural numbers: 37

c) whole numbers: 0, 37

d) rational numbers: $0.\overline{2}, 37, -\frac{4}{15}, -19, 8.51, 0$ Each of these numbers can be written as the quotient of two integers.

e) irrational numbers: $\sqrt{11}$, 6.149235... These numbers *cannot* be written as the quotient of two integers.

f) real numbers: All of the numbers in this set are real.
$\left\{0.\overline{2}, 37, -\frac{4}{15}, \sqrt{11}, -19, 8.51, 0, 6.149235...\right\}$

YOU TRY 3

Given the set of numbers $\left\{-38, 0, \sqrt{15}, 6, \frac{3}{2}, 5.4, 0.\overline{8}, 4.981162...\right\}$, list the

a) whole numbers b) integers c) rational numbers

d) irrational numbers

2 Compare Numbers Using Inequality Symbols

Let's review the inequality symbols.

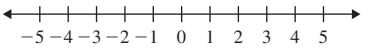
$<$ less than	$\leq$ less than or equal to
$>$ greater than	$\geq$ greater than or equal to
$\neq$ not equal to	$\approx$ approximately equal to

We use these symbols to compare numbers as in $5 > 2$, $6 \leq 17$, $4 \neq 9$, and so on. How do we compare negative numbers?

Hint
Think about why -8 is less than -7 on a number line. Draw a number line and notice that -8 is to the left of -7.

Note
As we move to the *left* on the number line, the numbers get smaller. As we move to the *right* on the number line, the numbers get larger.

EXAMPLE 4 Insert $>$ or $<$ to make the statement true. Look at the number line, if necessary.



a) $5 \_\_ 1$ b) $-4 \_\_ 3$ c) $-1 \_\_ -5$ d) $-5 \_\_ -2$

Solution

a) $5 > 1$ 5 is to the right of 1. b) $-4 < 3$ -4 is to the left of 3.

c) $-1 > -5$ -1 is to the right of -5. d) $-5 < -2$ -5 is to the left of -2.

YOU TRY 4 Insert $>$ or $<$ to make the statement true.

a) $4 \_\_ 9$ b) $6 \_\_ -8$ c) $-3 \_\_ -10$

We use signed numbers in everyday situations.

EXAMPLE 5 Use a signed number to represent the change in each situation.

a) After a storm passed through Kansas City, the temperature dropped 18°.

b) Between 2006 and 2010, retail sales of gluten-free products rose by $1.3 billion. (www.celiac.com)

Solution

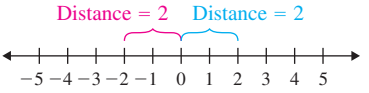
a) $-18°$ The negative number represents a decrease in temperature.

b) $1.3 billion The positive number represents an increase in sales.

YOU TRY 5 Use a signed number to represent the change.
After taking his last test, Julio raised his average by 3.5%.

3 Find the Absolute Value of a Number

Before we discuss absolute value, we will define *additive inverses*.



Notice that both -2 and 2 are a distance of 2 units from 0 but are on opposite sides of 0. We say that 2 and -2 are *additive inverses.*

Hint
Notice that the additive inverse of a negative number is always positive. For a positive number, the additive inverse is always negative.

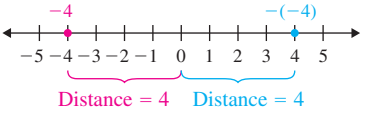
Definition

Two numbers are **additive inverses** if they are the same distance from 0 on the number line but on the opposite sides of 0. Therefore, if a is any real number, then $-a$ is its additive inverse.

Furthermore, $-(-a) = a$. We can see this on the number line.

EXAMPLE 6 Find $-(-4)$.

Solution



Beginning with -4, the number on the opposite side of zero and 4 units away from zero is 4. So, $-(-4) = 4$.

YOU TRY 6 Find $-(-11)$.

This idea of "distance from zero" can be explained in another way: *absolute value*.

Definition

If a is any real number, then the **absolute value of *a*,** denoted by $|a|$, is

i) a if $a \geq 0$

ii) $-a$ if $a < 0$

Remember, $|a|$ is never negative.

Note
The absolute value of a number is the distance between that number and 0 on the number line. It just describes the distance, *not* what side of zero the number is on. Therefore, the absolute value of a number is always positive or zero.